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MATDIP301

Third Semester B.E. Degree Examination, Dec.2015/Jan.2016

Advanced Mathematics – I

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions.***

1 a. Express the following in the form a + ib,
$\frac{3}{1+i}-\frac{1}{2-i}+\frac{1}{1-i}$ and also find the conjugate. (06 Marks)

b. Show that $(a+ib)^n+(a-ib)^n=2(a^2+b^2)^{n/2}\cos(n\tan^{-1}(b/a))$. (07 Marks)

c. Find the fourth roots of $1-i\sqrt{3}$ and represent them on an argand plane. (07 Marks)

2 a. Find the n^{th} derivative of cos 2x cos 3x. (06 Marks)

b. If $y=e^{a\sin^{-1}x}$ then prove that $(1-x^2)y_{n+2}-(2n+1)xy_{n+1}-(n^2+a^2)y_n=0$. (07 Marks)

c. Find the n^{th} derivative of $\frac{x}{(x-1)(2x+3)}$. (07 Marks)

3 a. Find the angle between the radius vector and the tangent to the curve $r=a(1-\cos\theta)$ at the point $\theta=\frac{\pi}{3}$. (06 Marks)

b. Find the pedal equation to the curve $r=a(1+\cos\theta)$. (07 Marks)

c. Obtain the Maclaurin's series expansion of the function $e^x \sin x$. (07 Marks)

4 a. If $u=e^{x^3+y^3}$, then prove that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=3u\log u$. (06 Marks)

b. If $u=f\left(\frac{x}{y},\frac{y}{z},\frac{z}{x}\right)$, prove that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}+z\frac{\partial u}{\partial z}=0$. (07 Marks)

c. If $u=x^2+y^2+z^2$, $v=xy+yz+zx$, $w=x+y+z$, find $J\left(\frac{u,v,w}{x,y,z}\right)$. (07 Marks)

5 a. Obtain the reduction formula for $I_n=\int_0^{\pi/2}\cos^n x\,dx$ where n is a positive integer. (06 Marks)

b. Evaluate : $\int_0^{2a}\int_0^{\sqrt{2ax-x^2}} xy\,dy\,dx$. (07 Marks)

c. Evaluate : $\int_0^1\int_0^1\int_0^1 (x+y+z)\,dx\,dy\,dz$. (07 Marks)

6 a. Prove that $\beta(m,n) = \dfrac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}$. **(06 Marks)**

b. Evaluate: $\int_0^4 x^{3/2}(4-x)^{5/2}\,dx$. **(07 Marks)**

c. Evaluate: $\int_0^\infty x^6 e^{-3x}\,dx$. **(07 Marks)**

7 a. Solve: $\dfrac{dy}{dx} + x\sin 2y = x^3\cos^2 y$. **(06 Marks)**

b. Solve: $(e^y + y\cos xy)dx + (xe^y + x\cos xy)dy = 0$. **(07 Marks)**

c. Solve: $x^2 ydx - (x^3 + y^3)dy = 0$. **(07 Marks)**

8 a. Solve: $\dfrac{d^3y}{dx^3} - 6\dfrac{d^2y}{dx^2} + 11\dfrac{dy}{dx} - 6y = 0$. **(06 Marks)**

b. Solve : $(D^2 - 4)y = e^x + \sin 2x$. **(07 Marks)**

c. Solve : $(D^2 + D + 1)y = 1 + x + x^2$. **(07 Marks)**

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